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The Tumbling Cube and the Action of the Mind

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Abstract. It is shown that a very small change in the initial angle of orientation of a tumbling cube can be detected by a shift in the endpoint of the trajectory after the cube travels a minimum distance. Specifically, if a cube travels forward in the x direction, with s the number of forward steps (tumbles) in the trajectory, the average sideways deviation ΔY in final position produced by a change in initial angle dq_0 is given by $\Delta Y = a(s - s_0)$, where a is the average length of the sideways step during each tumble and s_0 a term which depends logarithmically on dq_0. It is proposed that the reason cubes (dice) are popular in games of chance is because this ability of the cube trajectory to magnify small changes allows the possibility for mental influence to act. A general outline is given for experiments using traveling cubes which can test for the action of mental influence.

Keywords: Tumbling cubes, Mental influence, Quantum randomness, Uncertainty principle

1.0 Introduction

Tumbling cubes (i.e., dice) are often used in games of chance, and it has been widely held over many centuries that a person can be "lucky" and influence the final position of the cubes. It was first shown by Evan Harris Walker (1975), in a rough analysis of the dynamics, that the trajectory of a traveling cube is extremely sensitive to any small influence which changes its angular orientation at the beginning of the trajectory. Therefore, as Walker proposed, if mental influence can produce a small effect at the beginning of the trajectory, it can change the cube's endpoint.

As noted above, Walker's analysis was very rough, and the purpose of this paper is to redo the dynamical analysis of the traveling cube, using a different and more detailed method. (For a comparison of Walker's analysis with this one, see Burns (2002b).) This more detailed analysis can provide a framework in which experiments can be designed to test the possibility that a traveling cube can be affected by mental intention. (No specific experimental designs are suggested here, but a general outline is given.)

It should be pointed out that Walker's original analysis was made in the context of a larger theory which proposed that mental influence acts within the limits of the uncertainty principle. The latter topic will be addressed in another paper (Burns, 2002b). For the sake of generality, no assumptions are made herein as to the magnitude of effects produced by mental influence -- it is only assumed that they are small.

With respect to the topic of mental action, we note that it is not presently known whether free will or any action of the mind exists. The neurophysiological functioning of the brain is not known in sufficient detail for it to be possible to determine whether free will exists by direct experimental measurements of brain processes (Burns, 1996). Furthermore, presently known physical laws encompass only determinism and quantum randomness, so if we mean by "free will" a process which is different from these, it can only be accounted for by a radical addition to these laws, which would describe the physical effects of this action of consciousness (Burns, 1999). However, a variety of models of consciousness have proposed that mind can act on matter and have explored the means by which this action may relate to presently known physical laws, such as emergence, collapse of the wave function, or ordering of quantum fluctuations, some recent examples being Burns (2002a), Mohrhoff (1999), Mould (1998, 1999), Sirag (1993, 1996), and Stapp (1993, 1999). These models show that it is at least possible for a radical addition describing the physical effects of consciousness to be compatible with presently known laws.

With respect to the possibility that mind can have a physical effect outside the brain, we note that the way in which mental action would occur outside the brain would presumably be similar to the way free will would occur within the brain. If free will occurs (presently unknown), it seems reasonable that mental action outside the brain could also occur on a limited scale. Indeed, because of our present lack of detailed knowledge of the complex biochemical processes inside the brain, it might be easier to test for the existence of mental action, and explore its properties if found, outside the brain rather than within it.

The organization of the rest of this paper is as follows. In Section 2 we will note that a cube may travel forward either by tumbling (with one corner always touching the surface the cube travels on) or by bouncing (in which the cube has a substantial vertical component of velocity and spends most of its time in the air). For simplicity we will consider the case of the tumbling cube in this section and show that the analysis also applies to a bouncing cube later on.

The analysis of this section will show that a small change in the initial angular orientation of a tumbling cube can make a substantial change in its endpoint. Specifically, suppose that a cube makes s steps (tumbles about a corner) as it travels forward in the x direction. It is shown that if an influence produces a change dq_0 in the initial angle of orientation, the change ΔY in y coordinate at the end of the trajectory is given by $\Delta Y = a(s - s_0)$, where a is the average step taken by the cube in the $\pm y$ direction during each tumble and s_0 the minimum number of steps necessary before magnification can occur. We will also see that if an influence acts over the duration of the trajectory, nearly all the change ΔY is *produced* in the first few steps of the trajectory, with effects from later steps being very small.

In order to use the above formula, the average sideways and forward step lengths must be known. (The forward step length Δx must be known in order to determine the number of steps s the cube travels.) In Section 3 we will examine the dynamics of a tumbling cube and derive expressions for a and Δx. We will see that a considerable simplification is possible when the forward velocity is such that a parameter $z \approx 1$, because then a and Δx have a fixed ratio.

In Section 4 we will find the number of steps past s_0 which are necessary to have a tumbling cube end its trajectory with a given side face up.

Section 5 presents some additional details concerning the relationship between ΔY and X. We will note that random perturbations from air currents and surface irregularities produce a Gaussian envelope about the endpoint ΔY that would be obtained in the absence of these perturbations. Therefore, an experimental determination of ΔY involves comparing the midpoint of the Gaussian envelope obtained when there is a mental intention with the midpoint of the Gaussian envelope obtained with no such intention (or with an intention for the opposite direction).

In Section 6 we will see that the analysis of Section 2 also applies to a bouncing cube, and discuss further details of this mode of travel.

In Section 7 an estimate of s_0, based of Forwald's (1959, 1969) experiments, is given. In Section 8 further details are discussed relevant to doing experiments. Section 9 summarizes the conclusions herein.

2.0 The Deviation ΔY of a Tumbling Cube

A cube travels forward across a surface in the x direction. The z axis is normal to the surface, and the vector from the corner to the center of mass makes a skew angle to the z axis. (See Figure 1.)

A cube may travel either by tumbling or bouncing. If it is tumbling, it always keeps a corner in contact with the surface it travels on and rotates about that corner. If it is bouncing, it has a vertical component to its velocity and spends most of its time in the air. In the next several sections we will discuss the tumbling cube, and we will expand our discussion to include the bouncing cube in Section 6.

The motion of a tumbling cube is as follows. A corner is held in place by friction, and forward momentum causes the cube to rotate about that corner. During each forward step gravity produces a torque on the cube, and this causes the center of mass of the cube to rotate sideways -- to the right if the center of mass is to the right of the corner or to the left if the center of mass is to the left. Thus during each forward step the cube makes a sideways step in the $\pm y$ direction. Let a be the average length of this step.

We assume that the particular trajectory the cube follows depends on the initial angle q_0 between the vector from the corner to the center of mass and the z axis (with q_0 measured when the center of mass is in the y-z

plane). (See Figure 2.) Whether the corner is to the left or right of the *z* axis in a given tumble depends on the value of q at the beginning of the previous tumble and the magnitude of the forward and sideways rotations during that tumble. We assume that in a large number of runs the initial angle q_0 varies randomly and that trajectories with all possible combinations of left and right sideways steps occur, with each possible combination occurring in equal proportion. (The latter may not be strictly true, but should suffice for our analysis.)

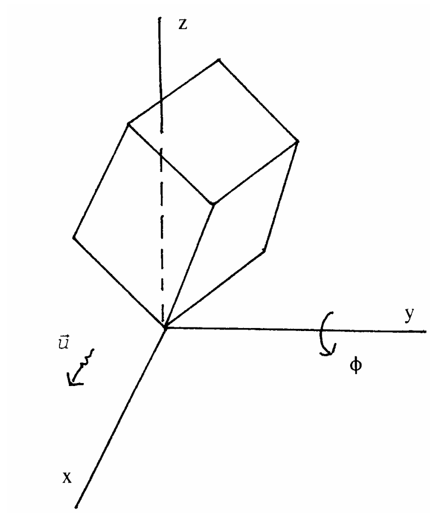

Figure 1. A cube tumbles forward along the *x* axis. Friction holds a corner in place, and the cube rotates about that corner by an average angle Δf during each tumble, such that the center of mass moves forward with average velocity *u*. The *z* axis is normal to the surface the cube tumbles on, which might be either horizontal (depicted here) or an inclined plane. The cube is oriented at a skew angle to the *z* axis.

Each trajectory is also subject to numerous small influences, such as surface irregularities and minute air fluctuations, which vary randomly from run to run. Thus we can think of each initial angle q_0 as being associated with a particular endpoint *Y* (where it would go if there were no random influences), plus an associated Gaussian distribution of endpoints caused by the random influences.

We now assume that at the beginning of each run the cube is also subject to a small influence which is the same in magnitude and direction each time. This influence might be either physical (such as a small puff of air) or, as is proposed here, a mental effect. This influence shifts the cube from one trajectory, with its associated value of q_0, to another trajectory characterized by an initial angle shifted by dq_0, and thereby shifts the original endpoint *Y* (with its associated Gaussian envelope) by ΔY .

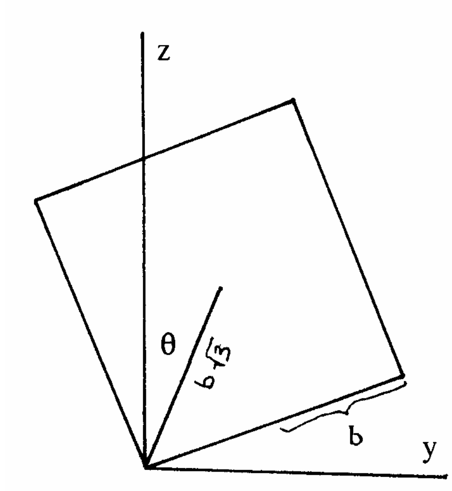

Figure 2. Projection of the cube on the *y*-*z* plane, at the time when the center of mass is in the *y*-*z* plane. The vector from the corner to the center of mass makes an angle q with the *z* axis. Each side of the cube has a half-length *b*, and the distance from the corner to the center of mass is $b\sqrt{3}$.

The set of all possible trajectories describing *Y* as a function of q_0 forms a continuum. However, for the sake of convenient analysis, we will represent them by a set of discrete trajectories. The number of possible trajectories n_{traj} is given by

$$n_{traj} = 2^s \ , \qquad (1)$$

where *s* is the number of forward steps. (See Figure 3.)

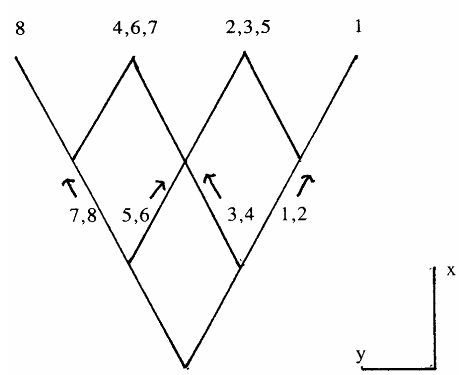

Figure 3. The continuum of all possible trajectories is approximated by a set of 2^S discrete trajectories, where *s* is the number of forward steps. The eight possible trajectories the cube can follow during three forward steps are shown.

q_0 varies over a range R_q, which equals p /3 by geometry of the cube. We assume that each discrete trajectory is equally probable. Therefore the spread in initial angle corresponding to each trajectory is $R_q / 2^S$.

We now ask the shift in endpoint ΔY which is produced by dq_0. Let us first assume that the change dq_0 takes place in a time t << t , the time for one tumble. (We will take up the more general case later.) Then the number of trajectories m which corresponds to dq_0 is given by

$$m = \frac{2^s}{R_q} dq_0 \; . \tag{2}$$

Suppose that dq_0 is sufficiently large that the final step in a trajectory can be changed, with all previous steps unchanged. Then two trajectories are involved, with a distance Δy between them of $2a$. Because all trajectories are equally probable, the average position of the original endpoint is at the center of this space. So the average change ΔY equals a.

Next suppose that dq_0 is sufficiently large that the final two steps in a trajectory can be changed. In that case $2^2 = 4$ trajectories are involved, and the range in endpoints corresponding to these trajectories is $4a$. The original endpoint would on the average be at the center of this range. So the average change ΔY is $2a$.

In general, to produce an average change in endpoint of ra, 2^r trajectories must be available for change. We set $\Delta Y = ra$ and $m = 2^r$. Therefore $\Delta Y = a \log_2 m$, and by equation (2) $\Delta Y = a[s - \log_2(R_q / dq_0)]$. We set R_q = p /3 and define s_0 by

$$s_0 = \log_2 \frac{p/3}{dq_0} \; . \tag{3}$$

We find

$$\Delta Y = a\,(s - s_0) \; . \tag{4}$$

We can understand the meaning of s_0 in the following way. By geometry of the cube, the range of angles available to q_0 is p /3. There are 2^S discrete trajectories, so dq_1, the spread in q_0 which corresponds to one discrete trajectory, equals (p /3)/2^S. When the trajectory has s_0 steps, dq_0 just fills dq_1. In that case dq_0 is not large enough to allow a change to a neighboring trajectory, so $\Delta Y = 0$. When the trajectory has more steps than s_0, dq_0 corresponds to more than one discrete trajectory, and ΔY can become greater than zero.

The above equations were derived for the case when the change dq_0 takes place at the beginning of the first step of the trajectory, with no change in q_0 thereafter. Let us now ask the result if q_0 changes throughout the trajectory. Let s' be the number of steps remaining in the trajectory at any given time. Then $dq_1(s')$, the angular range corresponding to a single discrete trajectory, is (p /3)/ $2^{s'}$. So if r trajectories are available for change during the first step, the total number r_{tot} available for change is $r[1 + 1/2 + (1/2)^2 + (1/2)^3 + ...]$, with the sum continuing for s steps. Therefore $r_{tot} < 2r$. But $r_{tot} = 2r$ is the same result that would be achieved if the number of steps were s+1 instead of s. Or, since ΔY is proportional to (s-s_0), this is equivalent to decreasing s_0 by one step. Therefore, if a change dq_0 during the first step produces a given value s_0, the cumulative effect of making a further change dq_0 per step during each additional step in the trajectory can be no more than to reduce s_0 by one step. The reason is that the number of trajectories which can fit in a given angular range decreases geometrically in each further step, so the effect of changes in q_0 during steps past the first quickly becomes very small.

The previous derivation also does not take into account the fact that even during the first step, the number of trajectories which can be affected is decreasing geometrically (from s to s-1). This also will be a small effect, but in the opposite direction to the above, i.e., it will tend to increase s_0 slightly. The two above effects tend to cancel each other. Thus for a continuing influence which causes a change $dq_0(t)$ in the first step (and for which the rotation per step does not change exponentially in succeeding steps), we can write

$$s_0 = \log_2 \frac{p/3}{dq_0(t)} . \qquad (5)$$

Thus even if an influence acts on the cube throughout its trajectory, most of the deviation ΔY is *produced* during the first few steps, with little contribution thereafter. On the other hand, the *magnification* results from having a sufficiently long trajectory, with $s > s_0$.

All dependence of ΔY on the rotation dq_0 is contained in the term s_0. And because s_0 depends logarithmically upon dq_0, its dependence on all factors contributing to dq_0 is logarithmic (unless any of those factors increases exponentially along the cube trajectory).

3.0 The Sideways and Forward Step Lengths

If we wish to use equation (4), we need to know *a* and *s*. To determine these experimentally, the average number of corners a cube tumbles about when it travels a given distance in the *y* direction (for *a*) and the *x* direction (for *s*) must be found. While high-speed movies could be taken, the problem in making these measurements is that the view of the corner the cube tumbles on would often be obscured and it would be difficult to tell when the cube had changed from one corner to another. However, we can analyze these theoretically. We will find that if a certain parameter z is about equal to one, the sideways and forward step lengths have a constant ratio, a very convenient relationship which simplifies the use of equation (4).

Let us start our analysis by setting forth some basic relationships. As specified in Section 2, a cube tumbles forward in the *x* direction across a surface, which might be either horizontal or an inclined plane (i.e., a ramp). The *x* and *y* axes are parallel to the surface (with the *x* axis pointing down the ramp), and the *z* axis is normal to the surface.

During each tumble, the cube rotates through an angle Δf about the *y* axis. E_f, the portion of the kinetic energy due to motion in the *xz* plane, is given by $E_f = 1/2\ I_f\ w_f^2$, where w_f is the angular velocity about the *y* axis and I_f is the associated moment of inertia about the corner. But $E_f = 1/2\ M(u^2+v^2)$, where *u* is the *x* component of the velocity of the center of mass, and *v* is its *z* component. The term involving v^2 is due to a small displacement of the center of mass as the cube rotates, and can be neglected. We take *u* to be the average value of the forward velocity, and find

$$w_f = u\,(M/I_f)^{1/2} . \qquad (6)$$

The moment of inertia of a cube about an axis through its center of mass is proportional to Mb^2, where *b* is the *half-length* of a side; the constant of proportionality is a number near unity, the exact value depending on the orientation of the axis and the distribution of density (e.g., whether the cube is solid or hollow). To evaluate the moment of inertia about a corner, we assume that the center of mass is directly above the corner the cube is rotating about, and neglect variations due to rotations in f and q because, by the geometry of the cube, neither angle can vary more than 30° from the vertical. We similarly assume that the center of mass moments of inertia about the *x* and *y* axes are the same. The values of I_f and I_q, the corresponding moments of inertia about an axis through the corner of the cube, can be found from the parallel axis theorem (Goldstein, 1980): the term $3Mb^2$ must be added to each center of mass moment because the corner is a distance $b\sqrt{3}$ from the center of mass. We have

$$I_f = I_q\ ; \qquad (7)$$

$$I_q = 4Mb^2 g \ . \tag{8}$$

where g is a factor close to one.

A cube with its center of mass at a skew angle to vertical will tumble about six different corners in a full rotation of $360°$. Hence on each tumble, the cube rotates an average angle Δf of $60°$. We will take the effect of sideways rotation into account in the next subsection. We wish to know the time t_f the cube takes to rotate through an angle Δf of $60°$. We have

$$\Delta f = w_f t_f = p/3 . \tag{9}$$

Evaluating t_f using equations (6), (7), (8) and (9), we find

$$t_f = \frac{2p}{3}\frac{b}{u} g^{1/2} \ . \tag{10}$$

We will see in the next subsection that when the cube is slowing down near the end of its trajectory, the time for the next successive corner to reach the surface is determined by the sideways rotation, not the forward motion. In that case the time between successive tumbles will be less than the time t_f for the cube to rotate forward by $60°$. We will compute the time between successive tumbles in the next subsection.

3.1 The Sideways Step Length

During each tumble the center of mass of the cube travels a sideways distance *a* because of the rotation produced by gravity. Let q be the angle between the vector from the corner to the center of mass and the *z* axis. (We measure q when the center of mass is in the *yz* plane and neglect the small variation in q which occurs during a forward tumble in the absence of sideways rotation.) The projection *y* of that vector on the *y* axis, and its differential change *dy*, are given by (see Figure 2)

$$y = b\sqrt{3}\sin q \ ; \tag{11}$$

$$dy = b\sqrt{3}\cos q \ dq \ . \tag{12}$$

Gravity produces a torque *L* which acts from the corner to the center of mass; we have

$$L = Mg\cos i \ y = I_q \frac{d^2 q}{dt^2} \ , \tag{13}$$

where g is the acceleration of gravity and i is the inclination of the plane (ramp) the cube travels on. We set $\sin q = q$, because q can be no greater than $30°$, by geometry of the cube. Then

$$\frac{d^2 q}{dt^2} = Cq \ , \tag{14}$$

where

$$C = \frac{b\sqrt{3}\,Mg\cos i}{I_q} \,. \qquad (15)$$

There are two solutions to equation (14), proportional to $\exp(C^{1/2}t)$ and $\exp(-C^{1/2}t)$, respectively. The solution $\exp(-C^{1/2}t)$ describes the case in which the sideways deviation is decreasing instead of increasing as the cube makes its irregular sideways path. However, we are only interested in the net sideways deviation, not the back and forth motion, so we do not need the latter solution. So we have

$$q = q_0 \exp(C^{1/2}t) \,. \qquad (16)$$

We set the initial angle q_0 equal to its average value. q has a range from 0° to 30°, so we set $q_0 = p/12$.

In finding the sideways step length another consideration is involved. We noted earlier that in the absence of sideways rotation a cube tilted at a skew angle will make six tumbles in a rotation of 360°. If Δq, the angular deviation per step, is small, this will continue to be true, and we can find Δq by integrating the above equation from $t = 0$ to $t = t_f$. (If the net sideways deviation is n steps, we integrate to $n t_f$ and divide by n to find the average angular deviation per step.) However, if Δq is sufficiently large that q reaches $p/6$ during a tumble, then a new corner touches the surface because of the q rotation instead of the f rotation. We now wish to know the condition under which Δq reaches its maximum value, $p/6 - q_0 = p/12$, during a tumble.

Let us define a parameter z by setting $z\, t_f$ equal to the time it takes for the cube to rotate from q_0 to $p/6$, according to equation (16). We have

$$\frac{p}{12} = \int_0^{zt_f} dt \frac{dq}{dt} = q_0 C^{1/2} \int_0^{zt_f} dt \exp(C^{1/2}t) = q_0[\exp(zC^{1/2}t_f) - 1] \,. \qquad (16a)$$

Substituting $q_0 = p/12$, we find $\exp(z\, C^{1/2} t_f) = 2$. Therefore,

$$z = \frac{\ln 2}{C^{1/2} t_f} \,. \qquad (17)$$

By equations (8), (10) and (15),

$$C^{1/2} t_f = \frac{p\,(g\cos i)^{1/2}}{3^{3/4}} \frac{b^{1/2}}{u} \,. \qquad (18)$$

Therefore

$$z = \frac{u/62.2}{b^{1/2}(\cos i)^{1/2}} \,\mathrm{cm}^{1/2}/\mathrm{sec} \,. \qquad (19)$$

If $z \le 1$, then a new corner touches the surface because Δq, rather than Δf, has reached its maximum value. We write

$$\Delta q = p/12 \quad (z \le 1) \qquad (20)$$

To find the linear step length a we substitute increments for differentials in equation (12) and set $\cos q$ equal to its average value in the interval, $3/p$, to find

$$a = \frac{b\sqrt{3}}{4} \quad (z \le 1) \tag{21}$$

Let t be the time for one tumble. We have $t = t_f$ when $z \geq 1$ and $t = z\ t_f$ when $z < 1$. Using equation (10) and dropping a factor $g^{1/2} \approx 1$, we write

$$t = \frac{2p}{3}\frac{b}{u} \quad (z \ge 1) \tag{22}$$

3.2 The Forward Step Length

The distance from a corner to the center of mass, projected on the *xz* plane, is $b\sqrt{3}\cos q$. (The variation in *y* over the course of a step is neglected.) Therefore, Δx, the distance traveled forward by the center of mass during one step, is given by $2\,b\sqrt{3}\cos q\ \sin(\Delta f/2)$, where Δf is the angle the cube rotates in the *xz* plane as it tumbles about a single corner. (See Figure 4.) Setting $\sin(\Delta f/2) = \Delta f/2$, we have

$$\Delta x = b\sqrt{3}\cos q\ \Delta f \tag{23}$$

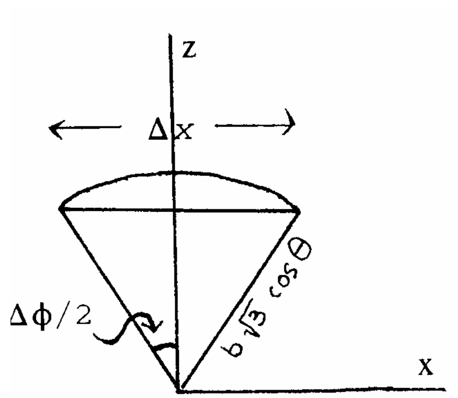

Figure 4. As the cube tumbles about a corner, the center of mass rotates by an angle Δf about the *y* axis (perpendicular to the page) and thereby travels forward a distance Δx along the *x* axis.

As discussed above, the value of Δf depends on a parameter z . At the beginning of the trajectory, when $z \geq 1$, $\Delta f = p/3$. However, as the forward velocity *u* becomes small through frictional losses, the increased time for each tumble allows the cube to rotate more in the *yz* plane. After the latter rotation reaches its maximum value, $p/12$, then $\Delta f = z\ p/3$. Setting $\cos q$ equal to its average value in the interval, $3/p$, we write

$$\Delta x = b\sqrt{3} \quad (z \ge 1) \tag{24}$$

3.3 The Ratio of Step Lengths When $z = 1$

If the cube tumbles down a gently sloping ramp such that $z = 1$ over much of its trajectory, then by equations (21) and (24)

$$\frac{\Delta x}{a} = 4 \quad (z = 1) \tag{25}$$

4.0 Mental Intention for a Given Side of a Cube to End Face Up

If a person is gambling with dice, he or she intends that a given ("target") side of the cube be face up at the end of a trajectory. Let us therefore ask the number of steps past s_0 in which the odds for ending with a given side face up are substantially improved.

The conditions to ensure that a target side end face up are more complex than those to simply ensure that the cube be cumulatively deviated in a given direction because the cube can stop traveling for several different reasons. In the first place, friction gradually decreases its velocity. So if no other factor comes into play, eventually there will be insufficient velocity to rotate the center of mass about a corner and the cube will stop. On the other hand, if two corners reach the surface at about the same time, and the velocity is orthogonal to the edge between these corners. the cube will start to roll. It will then come to a stop from a rolling motion, not from tumbling. And finally, if two corners reach the surface at about the same time, and the velocity is not sufficiently orthogonal to the edge between the corners, the cube will stop traveling. So the side which ends face up depends on details of its orientation and direction of travel during the final steps of the trajectory, and not merely on its sideways deviation.

Nevertheless, as we have seen in the previous section, each sideways step corresponds to a rotation of the center of mass about a corner, so a net sideways deviation of a sufficient number of steps will produce a sufficient cumulative rotation for the cube to end with a different side face up. As noted earlier, a cube oriented at a skew angle tumbles about six corners in a full rotation. Therefore, a sideways rotation of $\pm p/3$ ensures that two additional faces can be accessed. We ask the number of steps f past s_0 necessary to produce this rotation.

Equation (4) expresses the deviation ΔY of the cube in terms of linear steps of length a. However, we can equivalently define $\Delta\Theta$ to be the total angular deviation, and noting that Δq is the angular deviation per step, we write $\Delta\Theta = \Delta q\,(s - s_0)$. We then have

$$\Delta\Theta = \Delta q\, f = p/3\,. \tag{26}$$

In general, we do not know the value of a or Δq. However, if $z \approx 1$, then $\Delta q = p/12$ (equation (20). In that case $f = 4$.

Thus for the case when $z \approx 1$, the probability that a given ("target") side ends face up is substantially increased in only four steps past s_0. Even though three sides out of the six can potentially be accessed, we cannot say that the probability of ending with the target face reaches 1/2 for two reasons: (a) as described above, having a given side end face up involves more complexity than simply producing a cumulative deviation, and (b) as noted in Section 2 and to be discussed more fully in the next section, various fluctuations such as air currents produce a Gaussian envelope about each unperturbed endpoint, and these random fluctuations would prevent full accessing of other target sides.

5.0 Further Details About the Deviation ΔY

In this section we will discuss further details on the relationship between ΔY and X, the forward distance traveled. Noting that $X = s\Delta x$, let us rewrite equation (4) in the form

$$\frac{\Delta x}{a}\Delta Y = X - s_0 \Delta x\,. \tag{27}$$

If we were to plot a series of measurements of ΔY against X, we would want to have the line rise with a 45° slope, for ease in recognizing the point $\Delta x\, s_0$, and that is the reason for writing the equation in that form.

Obviously, therefore, it is useful to know the values of Δx and $\Delta x/a$. In general, these are not well known theoretically, although they can be estimated. (See example in Section 7.) However, in the case that the parameter z (equation (19)) = 1, we know from equations (24) and (25) that $\Delta x = b\sqrt{3}$ and $\Delta x/a = 4$. In that case we can write

$$4\Delta Y = (X - b\sqrt{3}\, s_0) \quad (z = 1) \tag{28}$$

This equation is plotted as the dashed line in Figure 5.

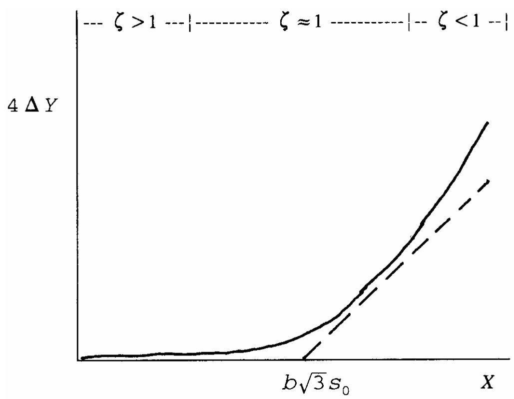

Figure 5. The dashed line shows $4\Delta Y$ as a function of X as predicted by equation (28). The solid curve shows the gradual increase from $X = 0$ and the steepening of the curve near the end of the trajectory when further details are taken into account. The above curve is drawn for the particular case of a tumbling cube for which the parameter $z \approx 1$ over much of the trajectory. In that case X_0, where equation (28) begins its rise, equals $b\sqrt{3}\, s_0$, where b is the half-length of a side.

5.1 The Set of Trajectories Forms a Continuum, Not a Discrete Set

As noted in Section 2, equation (4) (and succeeding equations (27) and (28)) are based on an analysis in which the continuum of possible cube trajectories is represented by a set of 2^s discrete trajectories. This analysis only allows endpoints to change from one discrete trajectory to another and does not allow changes by a fraction of a trajectory. For that reason these equations predict that $\Delta Y = 0$ for $s < s_0$. In actuality the trajectories form a continuum and a change corresponding to a fractional shift in a discrete trajectory can occur. Therefore, as depicted by the solid line in Figure 5, a small but non-zero deviation can occur when s is less than s_0.

5.2 Increase in Sideways Step Length as the Forward Velocity Approaches Zero

Also, the derivation leading to equations (4), (27) and (28) used average values for the sideways and forward step lengths. However, as a cube slows down and comes to rest, the forward step length will necessarily drop to zero. We saw earlier that as the cube slows down in its forward travel, more time is available for it to rotate sideways during each step. So the forward step length will decrease more rapidly than the sideways step length, and the slope of the curve of $(\Delta x/a)\Delta Y$ versus X will increase. The solid curve in Figure 5 shows this increase in slope toward the end of the curve.

5.3 Each Endpoint is Associated with a Gaussian Envelope Due to Random Fluctuations

For a given initial angular orientation, a cube will have a specific endpoint *Y*. However, we assumed (in Section 2) that in a large number of trials the cubes will start with all possible angular orientations, and will thereby travel all possible 2^s discrete trajectories, with equal weighting to each one. For that reason the distribution of endpoints will form a Gaussian curve with root mean square width (standard deviation) $<Y^2>^{1/2} = s^{1/2}a$ (Haken, 1983). In addition, the cube will undergo a random assortment of perturbations from surface irregularities and air fluctuations, and as we have seen, such perturbations, especially during the first step, will affect the final position *Y*. Because these additional factors are random, they will not affect the above distribution. However, we must keep in mind that because the endpoints *Y* of the cube can only be viewed via this distribution, formed from all these random sources, the curve for ΔY in Figure 5 must be understood as the separation between the midpoints of two Gaussian distributions, the one that would be obtained without the effect of mental influence and the one that would be obtained when it is present.

Let us further note that the standard deviation of the Gaussian curve is apt to be comparable in size to the deviation ΔY produced by mental intention. Specifically, from equation (4) and the above expression for the standard deviation, their relationship is

$$\frac{\Delta Y}{<Y^2>^{1/2}} = \frac{(s - s_0)}{s^{1/2}} . \tag{29}$$

Therefore, a large number of trials must be made in any experiment, so the Gaussian curves corresponding to the shifted and unshifted conditions can be distinguished.

6.0 Bouncing Cubes

In Section 3 we analyzed the sideways and forward step lengths of a tumbling cube. In the latter type of travel the cube keeps a corner in contact with the surface it travels on, and the cube travels forward by a series of rotations (tumbles) about its corners. However, a cube can also travel forward by bouncing, i.e., with a substantial component of velocity normal to the surface it travels on, such that it is airborne most of the time. Let us now make a comparison of these types of travel.

First, we should note that the derivation of the equations in Section 2 applies equally to bouncing and tumbling cubes. All that is needed is for the cube to travel forward *s* steps, while taking a sideways step of average length *a* during each forward step, and that in a large number of trials in which the initial angle varies randomly, all of the 2^s possible trajectories occur, with each one occurring in equal proportion. Therefore, equations (3), (4) and (5), and accompanying discussion, are valid for bouncing cubes as well as tumbling cubes.

On the other hand, in a given distance *X* of forward travel bouncing cubes will probably produce less sideways deviation ΔY . The reason is that the forward step length will be longer. Therefore, *X* will include fewer steps. Furthermore, s_0 will be the same because it depends only on the angular shift in initial orientation. So if the forward step for a bouncing cube is twice as long as that for a tumbling cube, for instance, a bouncing cube will have to travel twice as far before reaching s_0.

These types of travel also differ in the source of the sideways step. A tumbling cube rotates sideways because a corner remains in place and gravity, acting on the center of mass, thereby produces a torque. A bouncing cube has no torque exerted on it during the time it spends in the air. In fact, in most experiments using bouncing cubes (Forwald, 1959, 1969; Radin & Ferrari, 1991) multiple cubes have been used which strike against each other and against adjacent walls, and the sideways steps come about because of these impacts. However, the sideways velocity comes at the expense of the forward component of the kinetic energy, and the cubes don't travel as far forward.

Realistically, any traveling cube is apt to both tumble and bounce over its trajectory, the relative degree of each depending on the particular setup used. If an experimenter is seeking to use the z = 1 condition described herein, it is preferable to use a ramp, let the cubes start from rest, and keep bouncing to a minimum. (For further discussion, see Section 8.) On the other hand, in Forwald's (1959, 1969) deviation experiments, which we will refer to in the next section, cubes were dropped from a height, tumbled and bounced down a short runway, and then traveled across a table top. Probably the cubes mostly tumbled in the latter part of these trajectories.

7.0 Estimated Value of s_0

Let us now inquire as to what previous experiments on the effects of mental influence on traveling cubes can tell us about the value of s_0. A number of experiments studying the effect of mental influence on dice have been done, with meta-analysis of these showing a statistically significant effect (Radin & Ferrari, 1991). However, in these experiments cubes were bouncing over relatively short trajectories, so results probably correspond to the part of the curve where $s < s_0$.

On the other hand, a Swedish engineer, Haakon Forwald, conducted numerous cube deviation experiments a number of years ago (Forwald, 1959, 1969). Forwald did not know that a cube trajectory is extremely sensitive to air pressure and consequently took no precautions regarding the influence of breath or hand motions. (He stood next to the table the cubes tumbled on while focusing on his intention.) However, he got significant results in deviation experiments using a fairly long trajectory, which *may* have been due to mental influence, so his results can at least provide a tentative value of s_0.

In Forwald's experiments cubes were allowed to follow their trajectories until they naturally came to rest. The cubes typically traveled a distance of 15 cm down a ramp (McConnell & Forwald, 1968) plus 35 cm across a horizontal surface, for a total of 50 cm. The cubes had a half-length b of 0.8 cm. The average deviation ΔY was typically about 5 cm. This effect seems big enough that he would have been working in the range where $s > s_0$.

The cubes were dropped onto the ramp and undoubtedly bounced as they were traveling down it. However, they probably mostly tumbled as they traveled across the tabletop. The value for z will help us estimate the step lengths. Using equation (19) and the above value for b, we have z = u/55.6. Forwald (1959, 1969) gave the forward velocity u_0 when the cubes reached the tabletop as 186 cm/sec. (The latter was computed from the height the cubes traveled down the ramp.) Therefore, the cubes started across the tabletop with z greater than 3. This means that for most of their travel the forward step Δx was $b\sqrt{3}$. We will assume that the longer steps down the ramp, when the cubes were bouncing, were approximately balanced by the shorter steps after they dropped below 55.6 cm/sec, and take the average value of Δx as $b\sqrt{3}$. We note that $X = s\,\Delta x$ and substituting X = 50 cm, find $s \approx 36$ steps.

Let us now estimate s_0. We write equation (27) in the form $s_0 \Delta x = X - (\Delta x/a)\Delta Y$. If the cubes had gotten their sideways velocity only from tumbling, and z ranged around 1, $\Delta x/a$ would range around 4 (equation (25)). However, the cubes were striking against each other as they traveled on the ramp, and they gained energy in their sideways motion that way. Therefore, we estimate $\Delta x/a = 2$. Values of this ratio of either 4 or 1 seem unlikely (the latter would have the cubes traveling at a 45° angle to their initial direction during each step), and we take the range to be 1.5 to 2.5. From this range and midpoint we find (in round numbers) that $s_0 = 29 \pm 2$. The forward distance of travel, X_0, corresponding to these values of s_0 is (in round numbers) 40 ± 3 cm.

As a matter of interest let us also compute $\Delta Y / \langle Y^2 \rangle^{1/2}$. Using equation (29) we find $\Delta Y / \langle Y^2 \rangle^{1/2}$ = $(36-29)/36^{1/2} = 1.17$. So the shift ΔY was about the same as the standard deviation of the original distribution.

8.0 Experimental Considerations

According to equation (4) and the discussion of Section 5, the action of mental influence on a traveling cube will produce an average sideways deviation which is small until the cube has traveled a certain number (s_0) of forward steps, shows substantial increase at that point, and increases with distance thereafter.

In order to test this prediction a set of measurements can be made at varying travel distances X. The cubes could be filmed by an overhead camera and measurements of ΔY made at a set of distances along each trajectory.

Compensation can be made for possible right-left bias in the following way. In Part A of each experimental set a certain number of trials are accompanied by the mental intention that the cube will be deflected to the right, and in Part B the same number of trials are accompanied by the mental intention that it will be deflected to the left. The average deviation ΔY then is one half the difference between the average Y coordinate for the B readings and the average Y coordinate for the A readings. (This is the method that Forwald (1959, 1969) used.)

In designing an experiment the following points should be taken into account.

1. As we have seen, the sideways deviation ΔY is extremely sensitive to perturbations during the first few steps of the trajectory. Thus in order to test for the effects of mental intention, the cube must be shielded from any

air currents which might be correlated to intention, such as those produced by breath or hand movements. (Unfortunately, it was not previously recognized that a cube trajectory could produce this extreme magnification, and previous parapsychology experiments with traveling cubes have not taken precautions for this.)

2. As discussed at the beginning of Section 3, it would be difficult to determine the forward and sideways step lengths experimentally. However, we have seen that if the forward velocity of a tumbling cube is such that the quantity z (equation (19)) ≈ 1, then the average values of these quantities have a ratio of 4 (equation (25)). For that reason it will probably be more convenient to have cubes tumble down a gently sloping ramp and use this relationship. An additional convenience of using a ramp is that the acceleration imparted from it will help keep the cubes tumbling for a greater distance.

3. Realistically, as cubes tumble, they are also apt to bounce and skip to a certain extent. However, if the motion is primarily tumbling and the step length during bounces is not much greater than that during tumbling, this should suffice for the above relationship.

4. The coefficient of friction between the cubes and the surface they travel on should be sufficient that the cubes will tumble rather than slide. (Forwald had his cubes travel on a surface of smooth woodfiber board with a sliding coefficient of friction of 0.4. For discussion of this and further details of Forwald's experimental setup, see McConnell & Forwald (1968).)

5. Cube trajectories have certain idiosyncrasies. If two corners of a cube reach the surface at about the same time, and the direction of motion is orthogonal to the edge, the cube will start to roll instead of tumble. If two corners reach the surface at about the same time, and the direction of motion is not orthogonal to the edge, the cube will come to a dead stop, a phenomenon often seen. At the end of the trajectory, when the forward velocity is very small, a cube will sometimes make continued sideways steps in the same direction and trace out a semi-circle. Strictly speaking, the latter displacements should not be counted in determining ΔY, as this motion was not taken into account in the analysis.

6. Multiple cubes can be used in each experimental trial. However, if the above condition ($z \approx 1$) is used, then tumbling cubes should not be allowed to strike each other (or anything else), as this action would change the average sideways step length and was not provided for in the analysis. (See Burns (2002b) for further discussion on the use of multiple cubes.)

7. It should be noted that Forwald felt that he could produce much better results when he did no more than one set (10 cube releases) per day (Forwald, 1969). Researchers exploring other mental effects, such as remote viewing, also report that better results are obtained when only one experiment is done per day (Targ & Katra, 1997), so experimenters studying mental influence on traveling cubes may want to use the same policy.

8. It is likely that individual persons differ in their ability to affect an object through mental influence (Broughton, 1991). Therefore, different people will produce somewhat different values of s_0 for a traveling cube, and such differences will smear out the graph in Figure 5. For that reason, it is preferable to use a relatively small pool of subjects who may be reasonably supposed to have comparable abilities in producing mental influence.

9.0 Summary

It is not presently known whether mental influence exists, or if it does, whether it is inherently small and must be magnified to produce effects at the macroscopic level or whether it can act directly at that level. However, in this article we have supposed that mental influence is an inherently small effect and asked how it could affect the motion of a traveling cube.

We have seen that a traveling cube can greatly magnify a small change in angular orientation in the beginning of its trajectory, to produce a macroscopic shift sideways at the end of it. Thus if mental influence is inherently small, it can take advantage of this exceptional magnifying effect to produce a macroscopic change, and that may be the reason that tumbling cubes (dice) have been popular in games of chance for many centuries.

Specifically, we have seen that if a cube is traveling forward in the x direction, the shift ΔY in the average sideways position of the endpoint equals $a(s-s_0)$, where a is the average sideways step length, s is the number of steps (changes from one corner to another) in the trajectory, and s_0 depends logarithmically on the angular change. Nearly all of the shift ΔY is *produced* by changes in the angle of orientation during the first few steps in the trajectory, with changes in this angle in further steps being unimportant. However, it is necessary to travel a minimum number of steps s_0 before any substantial amount of *magnification* can take place.

It must be kept in mind that various random factors, such as air fluctuations, that the cube encounters in its first steps are also magnified. In the absence of these random factors, a cube starting with a given angular orientation would have an endpoint *Y* in the absence of mental intention, and would be shifted by an amount ΔY when mental intention is present. However, the random factors produce a Gaussian envelope in endpoints about *Y*, and similarly in the presence of mental intention produce a Gaussian envelope about the endpoint shifted by ΔY. Therefore, ΔY must be found experimentally as the difference between the midpoints of two Gaussian distributions of endpoints. Because the effects from random factors are apt to be as large, or larger, than those from mental intention, it follows that a large number of experimental trials are likely to be needed to establish the presence of mental influence.

It also follows from the tumbling cube's extreme ability to magnify initial changes that precautions must be taken to insulate the cubes from air currents from any breath or hand motions which might be correlated with mental intention for deviation in a given direction.

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References

[1] Broughton, R.S. (1991). *Parapsychology: The controversial science*. New York: Ballantine.
[2] Burns, J.E. (1996). The possibility of empirical test of hypotheses about consciousness. In S.R. Hameroff, A.W. Kaszniak, and A.C. Scott (Eds.), *Toward a scientific basis for consciousness* (pp. 739-742). Cambridge, MA: MIT Press.
[3] Burns, J.E. (1999). Volition and physical laws. *Journal of Consciousness Studies*, 6(10), 27-47.
[4] Burns, J.E. (2002a). Quantum fluctuations and the action of the mind. *Noetic Journal*, 3(4), 312-317.
[5] Burns, J.E. (2002b). The effect of ordered air molecules on a tumbling cube. *Noetic Journal*, 3(4), 330-339.
[6] Forwald, H. (1959). An experimental study suggesting a relationship between psychokinesis and nuclear conditions of matter. *Journal of Parapsychology*, 23, 97-125.
[7] Forwald, H. (1969). *Mind, matter, and gravitation*. New York: Parapsychology Foundation. (This booklet is still in print and can be obtained from Parapsychology Foundation, 29 West 57th St., New York, NY 10019.)
[8] Goldstein, H. (1980). *Classical mechanics*, 2nd ed. (pp. 196-198). Reading, MA: Addison-Wesley.
[9] Haken, H. (1983). *Synergetics*. New York: Springer-Verlag.
[10] McConnell, R.A., and Forwald, H. (1968). Psychokinetic placement: III. Cube-releasing devices. *Journal of Parapsychology*, 32, 9-38.
[11] Mohrhoff, U. (1999). The physics of interactionism. *Journal of Consciousness Studies*, 6(8-9), 165-184.
[12] Mould, P.A. (1998). Consciousness and quantum mechanics. *Foundations of Physics*, 28(11), 1703-1718.
[13] Mould, P.A. (1999). Quantum consciousness. *Foundations of Physics*, 29(12), 1951-1961.
[14] Radin, D.I. and Ferrari, D.C. (1991). Effects of consciousness on the fall of dice: A meta-analysis. *Journal of Scientific Exploration*, 5, 61-83.
[15] Sirag, S.-P. (1993). Consciousness -- A hyperspace view. Appendix to J. Mishlove, *Roots of Consciousness*, 2nd ed. (pp. 327-365). Tulsa, OK: Council Oak Books.
[16] Sirag, S.-P. (1996). A mathematical strategy for a theory of consciousness. In S.R. Hameroff, A.W. Kaszniak, and A.C. Scott (Eds.), *Toward a scientific basis for consciousness* (pp. 579-588). Cambridge, MA: MIT Press.
[17] Stapp, H.P. (1993). *Mind, matter and quantum mechanics*. New York: Springer-Verlag.
[18] Stapp, H.P. (1999). Attention, intention and will in quantum physics. *Journal of Consciousness Studies*, 6(8-9), 143-164.
[19] Targ, R. and Katra, J. (1997). Psi-conducive protocols. *European Journal of Parapsychology*, 13, 95-95.
[20] Walker, E.H. (1975). Foundations of paraphysical and parapsychological phenomena. In L. Oteri (Ed.), *Quantum physics and parapsychology* (pp. 1-53). New York: Parapsychology Foundation.